Navigating Numberabad, the Numberland Wonderland

Working Paper - WP 49/2003 (1-12)

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THE INSTITUTE OF HEALTH SYSTEMS

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Printed in Hyderabad, 2003

IHS Library Cataloguing-in-Publication Data

Navigating Numberabad, the Numberland Wonderland: 2nd Edition\ Prasanta Mahapatra

ISBN 81 - 7934 - 23 - 6

Includes bibliographic references1. Mathematics 2. Decimal Numbers 3. Number SpaceI. Prasanta Mahapatra512.7 629-DDC 2001

Navigating Numberabad - the Numberland, Wonderland.

Prasanta Mahapatra¹

Hyderabad has been likened by some to a legendary city from the Arabian Nights. Imagine you have moved into this unique Indian, cosmopolis. You plan to live here for years. You have a dream to fulfill and a plan to pursue. What do you do as you settle in to this new city? One thing that most people would do is to familiarise with the city. You explore the main streets and by lanes of the city that is your new home. Some would take a map, study it and systematically explore the city. Some may just wander around the city many a times and get feel of it. You walk around your residential area. Some travel by the bus and some use the ubiquitous Hyderabadi 'auto' to navigate the city. Some would bicycle down its lanes and by lanes, as I did in 1981, when I first moved into Hyderabad. The key step that most of us consider when we move into a new city and plan to build a career is to familiarise with its lanes and by lanes, recognise important land marks and make sure that we can find our way to important service locations.

Now think of yet another legendary from the Arabian Nights called the **Numberabad**. Numberabad is the number space. This is the Brindavan of mathematics, also called Numberland - the Wonderland. Public health, as you know is about people's health. That means, health of one and the many. Hence a public health worker has to count, compute, and calculate. All these arithmetic, algebra, and calculus, take place in the Numberabad. Many of us avoid a trip to this township. When the situation compels us to count or compute, we make quick and short visits using an easily available helicopter called the electronic calculator. If you were a mathematician, you had to live in Numberabad. As a public health worker, you can justifiably live in the Health Town and travel to Numberabad as and when required. Note however, that your work will require you to travel to this Number Town frequently. What do you do when your work takes you to the same city often? You familiarise with this business city so that you can easily find your way and are comfortable in places frequented by you. So it is important that you familiarise with the general topography (number space, Cartesian plane), directory (Number Sets), lanes (Number lines), and by lanes (interesting number relationships) of the Numberabad. You should be able to walk (add, subtract), bicycle (multiply or divide), hail an auto (raise the power of a number) to quickly get to your

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destination. If you hate the heat and dust of the auto and want to quickly get to your destination in comfort, you probably will hire a cab (logarithms and anti logs). Of course if you are the status conscious type and insist on getting around a small campus, in car (using logs and anti logs instead of simple multiplication), you will end up exhausted getting in and out of the car. In addition, your ceremonial entries and step outs from the car will probably amuse some people around. If you are visiting the Numberabad in a group and all of you want to navigate the town together then you will use the public transportation, I mean matrix algebra!

Now let us turn to the elements who live the Numberabad. People refer to a resident of Hyderabad, as a Hyderabadi. But a Numberabadi is called simply a Number. As you know, Hyderabad is home to a large variety of people from north, south, east and west. They do all sorts of work to earn a living. Now let us look at the different type of Numbers in Numberabad. What do the numbers do? Basically the Numbers are in, what economists call, the service sector². They serve human beings³. Of course, they were created by human beings to serve mankind. But it turns out that some animals do use number concepts at least to a limited extent (Devlin, 2000, p15-38).

² Economic activity has traditionally been classified by economists into the Primary sector, Industrial or manufacturing sector, and the Services sector. Primary sector consists of agriculture, animal husbandry, mining, all intricately linked with land. Industrial sector consisting of manufacturing activities such as making of steel (intermediate product) and consumer products such as bicycle, pressure cooker, etc. The Service sector consist of personal and business services. Services are some times referred to as intangible goods. In general services are consumed at the point of production. For example, you have to be there to benefit from the hair cut service rendered by a barber. The numbers are intangible but provide an important service to us to navigate the real world. Numerical activities happen within our heads and is consumed by our intellect. That justifies my analogy of numbers with the service sector. More recently some people have referred to a fourth sector, namely the Knowledge sector. My analogy of numbers in the service sector will hold with this addition as well. Numbers are not knowledge. Numbers enable knowledge and hence are playing a service role!

³ Well, does that mean that those of us who do not employ numbers are not human beings? Not really. After all, an agricultural labour, marginal farmer, small farmer, the regular farmer and big farmer are all human beings. But one works in others farm and another employs many people in her firm. That's the difference between those who can command many numbers into their service and those who have only a very few. But you see, many of us take time to realise this apparently simple insight. I was one of them. I did manage to live many years without too many of these number servants. Then, I thought, space is precious. If you have tried to rent an apartment or purchase a plot of land in Hyderabad you know how precious is space. So, I thought, why give these numbers so much space in my head. Instead, I could allot available space to clinical medicine, public administration, rural development, social welfare, health care or for that matter disaster management. It turns out that allocating more space to numbers and recruiting more of them to serve you actually increases your brain capacity to handle other things. I have learnt (better late than never!) that increasing familiarity with number space actually enhances one's ability to comprehend concepts and increased capacity to deal with day to day problems. In other words if you can think of the population of numbers serving you as a status symbol. The more numbers you have in your mental stable, the better off you are in the knowledge world. As of now, I am certainly not a Raja of numbers. Rather I am in the lower middle class, still working to improve my status.

We use numbers to express the ordering of things. Basically the numbers are rendering their ordinal property to let use express an kind of ordering. For example suppose we want to arrange a set of people based on the order of their arrival into this world. Members of this set of people may all enjoy the same status in society. So every one is equal in terms of their position and status. One example of such an ordering is the choice of protem speaker in legislative assemblies and our parliament. Our parliamentary tradition is to elect the eldest parliamentarian as the protem speaker of a newly constituted house. The protem speaker plays the role of the speaker until the election of the regular speaker. One way ancient people might have used number like expressions is to order the position of gods, say for example Brahma, Vishnu, and Maheswara. People might have used number like expressions to arrange the entry of characters into a drama in the desired order.

We use numbers to convey the size of a collection things, pressing into service the cardinal property of numbers. We measure the size of a collection of discrete objects simply by counting the number of such objects. Counting comes to us naturally. Hence mathematicians call the set of counting numbers $\{1,2,3,...\}$ as the Natural numbers. The symbol commonly used by mathematicians to represent the set of natural numbers is N. Recognising the fundamental position of natural numbers, Kronecker, a mathematician, is reported (Boyer and Merzbach, 1991, p569-70), to have said that "The natural numbers come from God and all else was man-made." We know that this is not a very accurate statement today. Nature has discrete objects and continuous objects. Nature has curves and undulations. Nature exhibits linear, wave and circular motion. We need the counting numbers to express magnitude of discrete objects. We need the other numbers to express various aspects of different natural phenomena. What this truly means is that natural numbers are closest to our senses. Note, however that counting in fact involves quite a few mental processes. First we identify discrete objects. It is relatively easy for our senses to tell one discrete object from another. But some situations may require special equipment. For example, a colony of bacteria grown on a petri dish looks like a continuous area. If we zoom in sufficiently with help of a microscope we can see each bacteria as a discrete object separate from other bacteria. We then classify various discrete objects by matching appropriate characteristics. For example, when you count the number of females and males attending a clinic, you are implicitly classifying people into two type of objects namely females and males. You are

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matching females with some notion of the characteristic of a female such as dress, name, appearance, and similarly matching males with some notion of male appearance⁴. If you want a little more detailed analysis of the attendance at a clinic, you might ask for counting of patients by the disease from which they suffer. Here doctors use the word diagnosis instead of matching. But in fact what a doctor does is to match the signs and symptoms of a given person with her⁵ knowledge of the pathophysiology and symptomatology of various diseases. After matching, you do sort or order the objects in the class to make counting feasible. Some times you do this physically, and some times you do it mentally. For example, when called upon to count the number of persons sitting in a room, we usually fix a ordering rule for ourselves, that I will start from the last row and proceed up row wise, or notionally divide the class room into two parts, count the people in first part and then continue with those in the second part. Pamela Liebeck (1984, p17-35) describes how children learn counting through these steps of matching, sorting, and ordering.

Once you have sorted the objects around you into relevant classes, you start counting the number of objects in each collection. Here Numberabad shares a problem with the Hyderabad. The city of Hyderabad has wards, neighbourhoods, streets, and houses. Each house usually goes by the name of the household that owns it. Some houses may have names given by their owners. For accurate identification a large city like Hyderabad needs a house numbering system. Hyderabad has tried many. The case of Numberabad is some what similar. Hyderabad has names and needed a system of numbers. Numberabad, instead, has numbers and needed a system of naming the numbers⁶. Either way the problem is similar. Hyderabad

⁴ I assume, that you are not considering a a medical examination to determine sex of the people attending a clinic. Medical examination protocol right upto genetic typing procedures exist for more accurate sex determination. But do not even think about it to count the number of males and females attending a clinic. It turns out that the dress, name, appearance, and for children the statement of the mother are robust enough protocols. The sensitivity and specificity of such conventional protocol is near one. We will learn about sensitivity and specificity in the epidemiology course. Suffice it to say that near one sensitivity combined with near one specificity means that you have a very highly accurate diagnostic tool.

⁵ I mean both him and her. I had sought comments from a colleague. One of the many 'errors' pointed out by my colleague was that I had mistakenly put 'her' instead of 'his'. This appears to be a result of gender stereotypes and technology stereotypes (Pam Linn, 1987). A public health worker needs to be wary of any such stereotypes. Gender discrimination is known to affect the health of women (Natta, 1999). Fortunately gender discrimination is being increasingly contested. Social scientists are working to identify the roots, and manifestation of gender discrimination. Social activists, are looking for policies to overcome systemic gender discrimination. Recently, a great deal of literature on gender relationships is appearing. A McNeil's (1987) collection of essays on Gender and Expertise, would be of interest to understand problems women face about their participation as scientists, experts and professionals. Frankly, I did not think much while putting in the 'her' in my script instead of the 'him' that some of my readers might expect. You see, I have a daughter, and my world revolves around her. ⁶ Although numbers needed a naming system, in turn numbers have also influenced the language, fascinated poets and philosophers. Nurnberg and Rosernblum (1968, p89-109) describe many English words with numerical connections and roots. For example; some of us compare millennium with million and ponder how is it that million is a thousand thousands but millennium is only a 1000 years! It turns out that millennium is

has tried with many numbering systems. The Numberabad has also experienced many a naming systems. Every number naming system has to have a base. If you do not have a base, you have to give an unique name to each and every number. You will need infinite names, which means that you can spend the rest of mankind's existence in the universe to just work out the names for the infinite numbers. But almost all of us would want to do other things in life. A base allows us to design a naming system for numbers taking advantage of the fact that one number can be added to another to create new one. For example, if we have a named numbers one and twenty, we can name (21=20+1) as twenty one. The smallest base is the binary consisting of two digits to represent one and two. Some people in South America, like the Bacairi and Bororo use the binary system for counting. In a way, most of us are using, the binary system now! We have used it in the computers. A computer's binary digits are off and on, which we translate to 0 and 1. A duodecimal system uses 12 as the base. In my child hood I came across both duodecimal system with base 12 and the hexadecimal system with a base of 16. I remember safety pins, cycle brushes, coming as in a dozen pack. A rupee had 16 anas. Each ana had four paisa, which is different from the paisa being used by us now. So the rupee had sixty four paise. India adopted the decimal system and we moved over to the modern rupee consisting of 100 paise. The rupee, of course, remained the same. We still retain the char (four) ana and ath (eight) ana to mean quarter, and half rupee respectively. Once again we are using the hexadecimal system or the hex codes in our computers. The Babylonians used a sexagesimal (sixty) base. We still retain it in our day to day life. This sexagesimal system explains division of the hour into sixty minutes, and division of the circle into 360 degrees. The vigesimal system had base 20. We now use the decimal system with base 10. But the words for numbers 11 to 19 have some carry over from the vigesimal system. That explains nineteen, which strictly would have been 'ten nine' under the decimal system similar to twenty nine, thirty nine etc. But the vigesimal system had separate names for numbers upto twenty. The decimal system has retained some of those names. The decimal system consists of 10 digits from 0 to 9, named zero, one, two, three, and so on till nine. We combine 10 to represent ten. The zero here acts as a place holder.

Order and size of collection of discrete entities can be expressed using the Natural numbers. But what happens if there is none of the object in question? The Hindus discovered the concept of Zero (Sunya) and the Arabs transmitted the idea to the rest of the world. Thus

derived from the Latin mille meaning a thousand. Hence a millennium is a thousand years. The word binomial meaning two names, two alternatives, or two categories, comes from 2.

mathematicians add the element Zero to the set of Natural numbers to get the set of Whole numbers. Yes you add Zero to make the Whole numbers from the Natural numbers. See how zero brings wholesomeness to naturalness! Strange but True!

The whole numbers are of three kinds, namely (a) Zero and One, (b) Prime Numbers and (c) Composite Numbers. Zero and One stand apart in a class of their own. Zero marks the starting point from which we count. Zero represents the Null set which is a set that does not have any element in it. The number 1 is mathematically the generator from which all other natural numbers are formed by successive addition. For example; 2=1+1, 3=2+1, 4=3+1, and so on. Zero is the additive identity. Thus zero added to or subtracted from any number leaves that number intact, i.e. $n \pm 0=n$ for all n. One is the multiplicative identity. This means that any number multiplied by One retains its identity and does not change into any other number. Division of a number by Zero destroys that number. In other words any number multiplied by zero, looses itself irretrievably. Division by zero is undefined. We simply do not divide some thing among none!

A Prime⁷ number is divisible by itself, one, <u>and none</u> <u>other</u>. An even number is by definition divisible by 2. An odd number is not divisible by 2. Thus all primes, except 2, are odd numbers. Primes are original numbers. These are the building blocks of all numbers. The primes are mathematically the material from which all other natural numbers are built up by multiplication (Hardy, 2002). There are infinitely many primes. Mankind is yet to discover any definite pattern to predict a prime number. In other words, we do not yet have a formula to generate a prime. The only

First 30 Prime Numbers			
2	31	73	
3	37	79	
5	41	83	
7	43	89	
11	47	97	
13	53	101	
17	59	103	
19	61	107	
23	67	113	
29	71	127	

⁷ We come across the word **Prime** in many fields with varying usage. In medicine a first time pregnant woman is referred to as primigravida. A pregnancy may terminate in abortion or parturition. Delivery of a viable fetus after 28 weeks of pregnancy is called parturition. A woman delivering for the first time is referred to as primipara. A woman delivering for the second or subsequent time is called multiprara. **Primipara** status has important implications for management of the delivery process. Obstetricians have to take extra precautions for a **primigravida** going into labour. In immunology, **primed** means an animal has prior contact with an antigen and hence subsequent challenge with the same antigen will result in a secondary response. Mechanical engineers refer to **priming of a pump** to the process of initial filling in of fluid to displace air before the pump is run for the first time. **Prime contract** in government and business means the concerned firm takes responsibility for the whole process of design, manufacture, test and supply, including management of sub contractors. In politics, as we see daily in the news papers, **Prime Minister** means the first and most important minister to aid and advise the President.

way to recognise a prime is to meticulously divide the number by all smaller numbers. The moment you find a whole number result of the division, you can stop, since now you know that the number is not a prime. Otherwise continue till you reach 1 as the divisor. Of course you will get the same number as a result of the division and you now know that the number is a prime. One thing we, however, know is that the prime numbers become sparser as we travel farther up on the number line. For example, there are 8 primes below 20 and four primes between 20-40. Primes have a tendency to crowd in pairs. For example; (11,13), (41, 43), (101,103). These are called twin primes. You see there are twins in the Numberland! Exercise caution. As it is difficult to tell if a woman will conceive a twin or single, baby, so is it difficult to tell where in Numberabad, you will come across a twin prime. You have to travel right up on the number line until you actually meet a pair of twin primes. Only then you know that they are the twin primes. Some mathematicians do spend time trying to discover larger primes and new twin primes.

Composite numbers are the product of some prime numbers. In other words composite numbers can be factored into some prime numbers. For example; $4 = 2 \cdot 2$, $6 = 2 \cdot 3$, $12 = 2 \cdot 2 \cdot 3$. Thus 4, 6, and 12 are composite numbers. The number 4 is composed of 2 times 2. The number 6 is composed of 3 times 2 or 2 times 3, etc.

Counting works when there is no need for accounting. If things are plenty, and contenders are few, then all you needed to do was to count your possession. But with scarcity comes accounting. You need some thing that is not available out there in the woods, but I have it. You need it and I have it. I give it to you on condition that you return it to me. That requires accounting! We have to maintain accounting from our respective perspectives. Hence the need for negative numbers. And you add these negative numbers to the set of whole numbers, to get the set of Integers {...-3,-2,-1,0,1,2,3...}, commonly represented by the symbol Z. An integers can be expressed as the sum or difference of two natural numbers. Without the negative numbers, you could only add natural numbers. Addition of the negative numbers to the set of Whole numbers makes subtraction possible for the full range of natural numbers. Now you see the link with accounting? After all accounting is about adding in one's account while subtracting from another account.

Fruits of individual efforts can be accounted for by Integers. Note that we are kind of literally talking about fruits which are discrete objects. We will proceed to the problem of measuring continuous entities in the following paragraph. For now let us stick to the discrete

world. Consider a company of individuals working together as a group to produce some things. How do they divide the fruits of their collective labour? The total produce of fruits is the dividend⁸ and the number of members in the group are the divisor. We now have to divide the dividend by the divisor to find out the share of each individual from the collective fruits of the company. For this we need what are called ratios. A ratio is one way of expressing relative size of two quantities, one divided by the other. Adding the English adjective ending 'al' to ratio, we have ratio-nal, which then came rational. The rational numbers are commonly represented by the symbol Q..

1	2	3	4	5	
1		3	4	5	
2		2	2	2	
1	2		4	5	
3	3		3	3	
1	2	3		_5_	
4	4	4		4	
1	2	3			
5	5	5	5		••••
• • • • • •					

A sam	ple of	Rational	Numbers
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The same word, namely rational, means reasonable, having or exercising the ability to reason. In philosophy, rationalism refers to a variety of views emphasizing the role or importance of reason in contrast to sensory experience (empiricism), introspection, feelings or authority (Lacey, 1995). Not a problem as along as the philosophy and mathematics do not cross path. In real life they do⁹. One saving grace is that the rational in mathematics is mostly

⁸ You guessed it right! The dividend that you earn from the share ownership of a company stock has the same root. The part of the profit that is set aside by a company for distribution among the share holders is called the dividend. The exact amount you get is based on the amount of the dividend, size of the divisor, i.e. total number of shares constituting the company's stock, and the proportion of total shares owned by you.

⁹ Bertrand Russell, for example, was a philosopher, mathematician, civil rights activist and literary figure. Russell pondered over Number theory and the nature of mathematics. His work Principia Mathematica (1910-13) jointly with Alfred North Whitehead, took up the issue of defining a natural number. For a brief

used together with the word numbers. More over people believe in reason or introspection. Numbers are creatures living in the number space created by people's mind. Numbers do not have a mind of their own. Thus rational numbers are ratio like numbers not reasonable numbers. A rational number can be expressed as a ratio of integers, except for zero in the denominator. In terms of decimals, a rational number may be a terminating decimal or a repeating decimal. Rational numbers make division operation possible. Note that the Rational numbers include the Integers, since every Integer can be expressed as the ratio of itself and 1.

Multiple	Prefix	Symbol	Common English Name (USA)	Some Indian	
10100				Tumber Tumes	
10 10 ²⁴	votto	v	bontillion		
10 10 ²¹	yotta	1	herillion		
10-2	zetta	L	nexillion		
1010	exa	E	quintillion		
10 ¹³	peta	Р _	quadrillion		
1012	tera	Т	trillion		
10^{9}	giga	G	billion		
10^{8}			100 million	Arab	
10^{7}			Ten million	Crore	
10 ⁶	mega	Μ	million		
105				Lakh	
10^{3}	kilo	Κ	thousand		
10 ²	hecto	h	hundred		
10 ¹	deca	da	ten		
10^{0}			One		
10-1	deci	d	tenth		
10-2	centi	c	hundreth		
10-3	milli	m	thousandth		
10-6	micro	Ü	millionth		
10-9	nano	n	billionth		
10-12	pico	р	trillionth		
10-15	femto	f	quadrillionth		
10-18	atto	a	quintillionth		
10-21	zepto	Z	hexillionth		
10-24	yocto	У	heptillionth		
¹ Source: Columns 2 -4 from CRC Hand Book of Mathematics, 1996 p5.					

Number Names in the Decimal System

overview of Russell's contributions to philosophy, see The Oxford Companion to Philosophy edited by Ted Honderich, 1995; p781-785.

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We are fine as long as we are doing business with discrete objects. But what about measuring things that are continuous, say for example, sides or area of a plot of land, the area of circle, the volume of a cylinder, etc.. It turns out that some of these measurements can not be expressed by any ratio. Take for example the diagonal of a right triangle with two sides equal to 1. The diagonal we know is $\frac{1}{2}$. We can approximate $\frac{1}{2}$ to any number of decimal places, but can not get a terminating or repeating decimal. Instead numbers like the †2, à, e, ln(2) give rise to non repeating decimals. Since these are not rational numbers, people named them irrational numbers. But as Deborah Hughes-Hallet (1980, p3) has observed, these numbers behave as reasonably well as the other numbers we have encountered so far. Another name of given to some of these is transcendental numbers. This, I think is a better name. Transcendental means some thing that is above ordinary human senses, some thing that transcends our sensory experience. We can relate Natural numbers to reality through counting. We can relate rational numbers through ratio of natural numbers or relationship of two magnitudes. The transcendental numbers require a little more imagination. They are a little more abstract. This does not mean that we do not experience these magnitudes. Beckmann (1971) narrates the history of how mankind has experienced the constant pi (à) through the circle ratio. Ratio of the circumference of any circle to its diameter is called the circle ratio. Many civilizations discovered this relationship almost independently but computed à to different degrees of approximations¹⁰. Although the initial discovery of à was through the circle ratio, it has since been found that area of many curves can be expressed in terms of à. The number à appears in probability theory very frequently.

The widely respected CRC¹¹ Mathematical Handbook (Zwillinger, Krantz, and Rosen, 1996) does not use the word irrational. Instead, it distinguishes between algebraic numbers

¹⁰ The ancient Babylonians estimated \dot{a} =3.125, ancient Egyptians estimated \dot{a} =3.16049..., the ancient Hindus estimated See Beckmann (1971) for a History of Pi.

¹¹I first heard about the CRC Handbook in while at Harvard between 1994-97. Every one seemed to know about it. But I was clue less as to what it stood for. Eventually I bought in June 1996, my copy of the book. I was disappointed to find that the book did not give any where an expansion of CRC which appeared to me to be an acronym. Every one was so familiar with this handbook that none was willing to engage in serious enough conversation with me about the book. However, I ventured to ask some of my teachers and colleague students. It took me quite a while to figure out that CRC stands for Chemical Rubber Company. It turns out that there is an interesting corporate history behind this. The following history of the CRC Press is based on a perusal on 14 Jan. 2003 of their web site at http://www.crcpress.com/corphistory.asp. The Chemical Rubber Company which is now known for publishing volumes of tabular information in the sciences and engineering got its origins as a company manufacturing rubber materials such as raincoats and laboratory coats. In those days laboratory coats were essentially oilcloth coated with a layer of natural rubber. As an advertising ploy in the early years of this century, the company began distributing with its rubber lab coats a small set of tables that included things like the common elements, atomic weights, and perhaps some math tables like logs. Over time the tables became larger and they became so useful that the company discovered that they could be sold independently of the lab coats. When modern plastic materials took over the lab wear business, the Chemical Rubber Company continued

(For example, the rational numbers and numbers like $\dagger 2$), and transcendental numbers such as the à and the *e*.

Thus the set of Real numbers (R) includes the Integers, the Rational and the Irrational or Transcendental Numbers. We use real numbers to measure continuous entities. Numbers are also used to model position and hence movement of things. Some of these modeling we can handle with the set of Real numbers. But for some we need to recruit a different kind of Numberabadi, called the imaginary numbers. Actually we are imagining the entire Numberabad. So it is a little absurd to name only a certain kind of Numberabadi as imaginary. The whole point is that these are relatively harder to visualise. We are asked to imagine the imagination of these! Thus the set of Complex numbers includes the Real numbers and the imaginary numbers.

As is the case with Hyderabad or for that matter in any human settlement on earth, Numberabad has its share of celebrities. Numberabad has its celebrities too. I have picked seven from the Clifford Pickover's (2001, p88-91) ranking, according to him of the ten most interesting numbers.

Seven Most Interesting Numbers

- 0 Important place holder. Enables faster calculations. Represents nothingness or null. See Charles Seife (2000) for a history of the development of the concept of zero. The concept of zero is also intricately linked with the concept of infinity. See Eli Maor (1991) for a history of the concept of infinity.
- à 3.1415... Non terminating Non repeating decimal. A transcendental number.
 Discovered as the circle ration, i.e. Ratio of the circumference of a circle and its diameter.
- *e* 2.7182... Non terminating Non repeating decimal. Transcendental number. Base of the natural system of logarithms. The limit of $(1+1/n)^n$ as *n* goes to infinity. Growth processes in biology such as that of bacteria, human population growth, are modeled by functions of the type $y = e^x$.
- *i* \neq -1. Imaginary unit. Defined as the solution to $x^2+1=0$. Used for spatially manipulating models of protein structure. The space shuttle's flight software uses it for navigation.
- *†*2 1.4142... Non terminating Non repeating decimal. Algebraic number.
- 1 A factor of all numbers. It is the multiplicative identity.
- 2 The only even prime number. Basis for the binary system.

in existence, but as the publisher of tables and not as a maker of lab coats. In 1973, the Chemical Rubber Company sold their manufacturing division and related activities. Concentrating exclusively on publishing, the name was changed to CRC Press, Inc. As of January 2003, the CRC Press is a publicly-held company under its current ownership of Information Holdings, Inc. It is located in Florida state of the USA. The 3M company shares a similar history and you probably have come across its products. 3M makes the post-it note pads, stationery items, floppy disks, etc. It started as the Minnesota, Mining and Manufacturing Company, failed in the business of mining, but succeeded in developing and marketing the world's first waterproof sand paper, and scotch adhesive tapes, etc.

By 2001 there were about 50 lakh people in Hyderabad. Yesterday's (14 Jan, 2003) advertisement by the Hyderabad Urban Development Authority says that we are planning the city for 150 lakh people. One difference in case of Numberabad, is that a census in Numberabad once started has to continue for ever. So we can not get a final tally of the total number of Numbers in Numberabad. Another difference is that all these numbers can live in no space. For example, the entire set of Real numbers can be represented in a line segment from 0 to 1. Another peculiarity of the Numberabad is that there is a hierarchy of the number sets that have infinite elements in it. Mathematicians consider the size i.e. Cardinality of the set of real numbers is bigger than the rational numbers.

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